QUIZ #4 – Solutions
Each problem is worth 5 points

15 points total

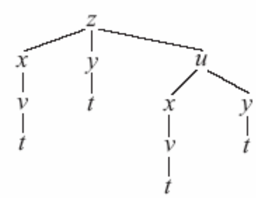
1.

In general,

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dv}\frac{dv}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial x}\frac{dx}{dv}\frac{dv}{dt} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\frac{dy}{dt} \\ &= \left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial x}\right)\frac{dx}{dv}\frac{dv}{dt} + \left(\frac{\partial z}{\partial y} + \frac{\partial z}{\partial u}\frac{\partial u}{\partial y}\right)\frac{dy}{dt},\end{aligned}$$

and specifically,

$$\begin{aligned}\frac{dz}{dt} &= \left\{2x + 2u\left[\frac{-2x}{(x^2-y^2)^2}\right]\right\}(3v^2-6v)e^t + \left\{2y + 2u\left[\frac{2y}{(x^2-y^2)^2}\right]\right\}4e^{4t} \\ &= 6xve^t(v-2)\left[1 - \frac{2u}{(x^2-y^2)^2}\right] + 8ye^{4t}\left[1 + \frac{2u}{(x^2-y^2)^2}\right].\end{aligned}$$



2.

Since a normal to the tangent plane is

$$\nabla(x - x^2 + y^3 z)_{|(2,-1,-2)} = (1-2x, 3y^2 z, y^3)_{|(2,-1,-2)} = (-3,-6,-1),$$

as is $(3,6,1)$, the equation of the tangent plane is

$$0 = (3,6,1)\cdot(x-2, y+1, z+2) = 3x + 6y + z + 2.$$

3.

For critical points we solve $0 = \dfrac{\partial f}{\partial x} = 3y - 3x^2,\ 0 = \dfrac{\partial f}{\partial y} = 3x - 3y^2$. Solutions are $(0,0)$ and $(1,1)$.

$$\frac{\partial^2 f}{\partial x^2} = -6x, \qquad \frac{\partial^2 f}{\partial x \partial y} = 3, \qquad \frac{\partial^2 f}{\partial y^2} = -6y$$

At $(0,0)$, $B^2 - AC = 9 - 0$, and therefore $(0,0)$ yields a saddle point. At $(1,1)$, $B^2 - AC = 9 - (-6)(-6) = -27$, and $A = -6$, and therefore $(1,1)$ gives a relative maximum.